Alpha-alpha interaction with chiral two pion exchange and ^{8}Be lifetime.

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Assuming that alpha particles are described by a scalar-isoscalar chiral invariant heavy field it is shown how chiral symmetry determines the alpha-alpha interaction at long distances unambiguously from dimensional power counting of an effective chiral Lagrangean. The leading strong contribution is given by a two pion exchange potential which turns out to be attractive and singular at the origin, hence demanding renormalization. When ^{8}Be is treated as a resonance state a model independent correlation between the Q-factor and lifetime $1/\Gamma$ for the decay into two alpha particles arises. For parameters compatible with potential model analyses of low energy $\pi\alpha$ scattering it is found a Breit-Wigner width $\Gamma = 4.3(3)$eV very close to the experimental value, $\Gamma_{\rm exp.} = 5.57(25)$eV.

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Low energy nuclear reactions in the energy range of astrophysical interest are generally extremely hard to measure experimentally in the laboratory [1]. This applies in particular to $\alpha\alpha$ scattering where ^{8}Be is produced as a narrow resonance [2]. In this paper we approach this problem from the theoretical side in the context of the chiral symmetry[3, 4] and Effective Field Theories (EFT) [5]. Specifically, it will be shown how the lifetime of ^{8}Be in alpha-decay may be accounted for in a model independent fashion by assuming an elementary field for the α-particle, and exploiting the spontaneous breakdown of chiral symmetry of QCD as well as the fact that the ^{8}Be ground state lies right above the $\alpha\alpha$ threshold. In addition to the standard Coulomb interaction, the scalar-isoscalar character of the α particle implies that the long distance strong interaction is dominated by two pion exchange (TPE) regardless on any specific internal structure.

Pion exchange interactions between α particles have been treated in the past in a variety of ways. A resonating group method approach was used in Ref. [6] with an approximation for the TPE in the mid-range. Forward dispersion relations for $\alpha\alpha$ scattering have been discussed [7]. A folding model from a NN potential was used in Ref. [8] and the I-wave phase shift was computed in first order perturbation theory. An EFT description of narrow resonances has been undertaken for pure contact theories [9], i.e. without pions or Coulomb forces.

The idea of associating elementary fields to nuclei at low energies is rather natural [10]. The α particle is a ^{4}He nucleus with $(J^P, T) = (0^+, 0)$, charge $Z_\alpha e = +2e$ and mass $M_\alpha = 2(M_p + M_n) - B = 3727.37$MeV, with binding energy $B = 28.2957$MeV. Because of the tensor force, the wave function for the ground state can be a positive-parity mixture of three $^1S_0$, six $^3P_0$, and five $^5D_0$ orthogonal states [11], the symmetric S-wave component being the dominant part of the wave function, with significant D-wave and almost negligible P-wave contributions. Thus, as a starting point we associate a scalar-isoscalar Klein-Gordon charged field, $\alpha(x)$ to the ^{4}He nucleus. Under charge conjugation the α-particle should transform into an anti-α-particle $\alpha(x) \to \bar\alpha(x)$ meaning that the field is non-hermitean. Further, under $SU(2)_R \otimes SU(2)_L$ chiral transformations we assume $\alpha(x)$ to be invariant; candidates for chiral partners with the same $B = 4$ baryon number would be ^{4}H and ^{4}Li having both spin-2, $(J^P, T) = (2^-, 1)$, thus belonging to different Poincare group representations. The effective Lagrangean will include pions [3] and α particles which being much heavier, $M_\alpha \gg m_\pi$, are better treated by transforming the Klein-Gordon field as $\alpha(x) = e^{-iM_\alpha v\cdot x}\alpha_v(x)$ with $\alpha_v(x)$ the heavy field and v^μ a four-vector fulfilling $v^2 = 1$, eliminating the heavy mass term [12, 13]. Keeping the leading M_α term, the effective Lagrangean reads

$$
\begin{aligned}
\mathcal{L} \;=\;& iM_\alpha \bar\alpha_v v\cdot\partial\alpha_v + \frac{f^2}{4}\left[\langle \partial^\mu U^\dagger \partial_\mu U\rangle + \langle \chi U^\dagger + \chi^\dagger U\rangle\right] \\
&+\; g_0 \bar\alpha_v \alpha_v \langle \partial^\mu U^\dagger \partial_\mu U\rangle + g_1 \bar\alpha_v \alpha_v \langle \chi U^\dagger + \chi^\dagger U\rangle \\
&+\; g_2 \bar\alpha_v \alpha_v \langle v\cdot\partial U^\dagger v\cdot\partial U\rangle + \lambda\left(\bar\alpha_v\alpha_v\right)^2
\end{aligned}
\tag{1}
$$

where the pion field in the non-linear representation is written as a $SU(2)$-matrix, $U = e^{i\vec\tau\cdot\vec\pi/f}$, with $\vec\tau$ the isospin Pauli matrices, f the pion weak decay constant in the chiral limit $f = 88$MeV, $\chi = m^2/2$ and $\langle,\rangle$ means trace in isospin space. Here g_0, g_1, g_2 and λ are dimensionless coupling constants which are not fixed by chiral symmetry. This Lagrangean is the analog of the Weinberg-Tomozawa Lagrangean and EFT extensions for πN interactions [4, 13] to the case of the $\pi\alpha$ system. Photons are included by standard minimal coupling $\partial^\mu\alpha \to D^\mu\alpha = \partial^\mu\alpha + Z_\alpha e i A^\mu \alpha$. For definiteness we take, $M_\alpha = 3727.3$MeV, $f \to f_\pi = 92.4$MeV and $m \to m_\pi = 138$MeV.

To estimate the couplings g_0, g_1 and g_2 we look first at low energy $\pi\alpha$ scattering (see [4] for a review). From

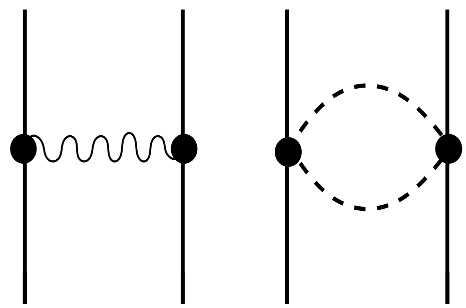

FIG. 1: Diagrams contributing to the $\alpha-\alpha$ potential at long distances: One-Photon Exchange (left) and Two-Pion Exchange loop (right). Full lines represent the α particle field, the dashed lines are pion fields, wiggly lines are photon fields. The full blobs are the $\alpha\alpha\gamma$ and the $\alpha\alpha\pi\pi$ vertices respectively.

Eq. (1) we get the $\pi^a(p_\pi)+\alpha \to \pi^b(p'_\pi)+\alpha$ invariant amplitude at lowest tree level order

$$iF^{ab}_{\pi\alpha\to\pi\alpha} = \frac{4i\delta^{ab}}{f_\pi^2}\left(g_0 p'_\pi\cdot p_\pi - g_1 m_\pi^2 + g_2 v\cdot p_\pi v\cdot p'_\pi\right) \quad (2)$$

where a and b are the final and initial pion isospin states respectively. In the limit $M_\alpha \gg m_\pi$ LAB and CM coincide, so taking $v^\mu=(1,\vec{0})$ and the kinematics as $p_\pi=(\omega,\vec{p})$ and $p'_\pi=(\omega,\vec{p}')$ with $\omega=\sqrt{\vec{p}^2+m^2}$ the scattering amplitude is given by ($\mathcal{F}=8\pi\sqrt{s}f$)

$$\begin{aligned} f_{\pi\alpha}(p,\theta) &= \frac{g_0(m_\pi^2+p^2(1-\cos\theta))-g_1m_\pi^2+g_2\omega^2}{2\pi M_\alpha f_\pi^2} \\ &= A_0+B_0p^2+3A_1p^2\cos\theta+\ldots \end{aligned} \quad (3)$$

where in the second line the threshold parameters A_0, B_0 and A_1 have been introduced. From mesic $\pi^- -{}^4$ He atoms one has $A_0=(-0.138+i0.045)$fm [4, 14] while $B_0=-0.18\text{fm}^3$ and $A_1=(0.42+i0.06)\text{fm}^3$ from forward dispersion relations [15]. Our description is not realistic concerning the direct comparison with data; Coulomb distorsion has been disregarded and treating the α particle as elementary precludes pion absorption since real $g_{0,1,2}$ imply $\text{Im}A_l=0$. However, realistic calculations allow to switch off these effects [16] yielding the pure potential values, $A_0^{\text{pot}}=-0.091(17)$fm and $A_1^{\text{pot}}=1.058(144)\text{fm}^3$. We take $B_0^{\text{pot}}=-0.2(2)\text{fm}^3$ from [15], the error being an educated guess, since there are unresolved discrepancies (see e.g. [17]). The small scattering length supports our perturbative calculation and in fact $A_0\to 0$ in the chiral limit, $m_\pi\to 0$. From the values above we get $g_0=-82(11)$, $g_1=-5.3(3)$ and $g_2=77(12)$. On the other hand, the double scattering contribution to the $\pi\alpha$ s-wave [18], yields the identification $g_1-g_0-g_2=M_\alpha\langle r^{-1}\rangle_\alpha/f^2$ which has the correct pion mass dependence, provided all quantities are evaluated in the chiral limit, and suggests that $g_1-g_0-g_2>0$. For $\langle r^{-1}\rangle_\alpha=0.5$fm, a realistic value, and using either f or f_π yields $g_1-g_0-g_2\sim 40-47$, while we get $\sim 0-20$ instead. A expected decrease of $\langle r^{-1}\rangle_\alpha$ in the chiral limit might naturally accomodate the discrepancy. On top of this, pion loop corrections to $\pi\alpha$ scattering, which are $\mathcal{O}(1/f_\pi^4)$, might have a sizable impact on g_0,g_1 and g_2 due to chiral logs. Clearly, a more systematic assessment of the input values of the couplings and their uncertainties would be most useful.

Let us now turn to the calculation of the long distance $\alpha\alpha$ potential. The leading direct t-channel TPE contribution is depicted in Fig. 1 and can be written as

$$\mathcal{F}_{\alpha\alpha\to\alpha\alpha}(q)=\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\frac{|\mathcal{F}^{ab}_{\pi\alpha\to\pi\alpha}|^2}{((p-q)^2-m_\pi^2)(p^2-m_\pi^2)}\,. \quad (4)$$

There is a factor $\delta^{ab}\delta_{ab}=3$ coming from $\pi^+\pi^-$ as well as $\pi^0\pi^0$ exchange (we neglect here tiny isospin breaking effects). As we see by power counting the integral is quartically divergent. Using the dispersion relation

$$\mathcal{F}(q^2)=\int_{4m_\pi^2}^\infty d\mu^2\frac{\rho(\mu^2)}{q^2-\mu^2+i0^+}+\text{c.t.}\,, \quad (5)$$

with $\rho(\mu^2)$ the spectral density and c.t. stands for counterterms, e.g. λ in Eq. (1), which will not contribute to the coordinate space potential at positive but nonvanishing distances. From Cutkosky's rules one gets

$$\rho(\mu^2)=\frac{1}{2\pi^2f_\pi^4}\left(A_-^2+2A_+^2\right)\left[1-\frac{4m_\pi^2}{\mu^2}\right]^{\frac{1}{2}}\,, \quad (6)$$

where $A_\pm=(g_1-g_0)m^2+g_0\mu^2/2\pm g_2(\mu^2/4-m^2)$. Note that $\rho(\mu^2)$ is a positive quantity. This result agrees with Ref. [8] only when $g_2=0$ and resembles a similar calculation for the central NN force [19]. In the heavy mass limit $\sqrt{s}\to 2M_\alpha$ and in the CM system $v\cdot q=0$ so that the potential is given by the expression

$$V^{2\pi}_{\alpha\alpha}(\vec{x})=-\frac{1}{4M_\alpha^2}\int\frac{d^3q}{(2\pi)^3}\mathcal{F}(-\vec{q}^2)e^{i\vec{x}\cdot\vec{q}}\,. \quad (7)$$

Computing the q and the μ integrals, up to distributions located at $\vec{x}=0$, the final result becomes

$$V^{2\pi}_{\alpha\alpha}(r) = -\frac{3m_\pi^7\left[K_0(2x)f(x)+K_1(2x)g(x)\right]}{32\pi^3M_\alpha^2f_\pi^4x^6} \quad (8)$$

where $x=m_\pi r$, $K_0(x)$ and $K_1(x)$ are modified Bessel functions and $f(x)=4(g_0+g_1)^2x^4+10(12g_0^2+4g_2g_0+3g_2^2)+\left[84g_0^2+24(g_1+g_2)g_0+g_2(4g_1+15g_2)\right]x^2$, $g(x)=4(g_0+g_1)(6g_0+g_2)x^3+10(12g_0^2+4g_0g_2+3g_2^2)x$. The TPE potential is attractive everywhere as can be recognized from the positivity of the spectral function. The potential (8) is the strong interaction analog of the time-honoured Casimir-Polder electromagnetic forces [20, 21]. In fact, the TPE potential becomes singular at short distances,

$$V^{2\pi}_{\alpha\alpha}(\vec{x})=-\frac{15(12g_0^2+4g_0g_2+3g_2^2)}{32\pi^3M_\alpha^2f_\pi^4}\frac{1}{r^7}+\ldots \quad (9)$$

This is a relativistic and attractive Van der Waals interaction which is explicitly independent on the pion mass. In the opposite limit of long distances we have

$$V^{2\pi}_{\alpha\alpha}(\vec{x}) \to -\frac{3(g_0+g_1)^2 m_\pi^{9/2}}{16\pi^{5/2} M_\alpha^2 f_\pi^4} \frac{e^{-2m_\pi r}}{r^{5/2}} \, . \tag{10}$$

Of course, the previous potential describes the strong interaction piece, and as we see is $\mathcal{O}(1/f_\pi^4)$. Electromagnetic effects correspond to minimally couple photons. We keep one photon exchange (see Fig. 1) but neglect two or higher photon exchanges as well as terms $\mathcal{O}(Z_\alpha^2 e^2/f^4)$ which may be systematically computed from higher dimensional corrections to the Lagrangean (1). These and other effects will be analyzed in more detail elsewhere.

The total potential in a long distance expansion is given by adding the TPE and Coulomb potentials

$$V(\vec{x}) = V^{2\pi}_{\alpha\alpha}(\vec{x}) + \frac{e^2 Z_\alpha^2}{r} + \dots \tag{11}$$

with $Z_\alpha = 2$ and $e^2 = 1/137.04$ the fine structure constant. The dots in Eq. (11) represent shorter range corrections than TPE. Rotational invariance allows to write the relative s-wave function as $\Psi(\vec{x}) = u_{0,p}(r)/\sqrt{4\pi}$ with $u_{0,p}(r)$ the reduced s-wave function fulfilling

$$-u''_{0,p}(r) \; + \; \left[U^{2\pi}_{\alpha\alpha}(r) + \frac{2}{a_B r} \right] u_{0,p}(r) = p^2 u_{0,p}(r) \, , \tag{12}$$

with $U^{2\pi}_{\alpha\alpha}(r) = M_\alpha V^{2\pi}_{\alpha\alpha}(r)$, $a_B = 2/(M_\alpha Z_\alpha^2 e^2) = 3.63$fm the Bohr radius and $p = \sqrt{M_\alpha E}$ the CM momentum. The problem is to solve Eq. (12) with suitable boundary conditions, but since the potential is singular and attractive at the origin, see Eq. (9), some renormalization proves necessary. Actually, the regularity condition, $u_{0,p}(0) = 0$, only fixes the solution up to an arbitrary constant [22, 23] which must be fixed *independently* of the potential. Furthermore, from the self-adjoint condition(for a discussion within the NN context see [24]) there is the relation

$$0 = \left[{u_{0,k}}^* u'_{0,p} - {u'_{0,k}}^* u_{0,p} \right] \Big|_{r_c} \tag{13}$$

where the limit $r_c \to 0$ for the short distance cut-off is understood. Taking $p = k$ and $k = 0$ we get further

$$\mathrm{Re}\left[\frac{u'_{0,p}(r_c)}{u_{0,p}(r_c)} \right] = \frac{u'_{0,p}(r_c)}{u_{0,p}(r_c)} = \frac{u'_{0,0}(r_c)}{u_{0,0}(r_c)} \, . \tag{14}$$

This means that the logarithmic derivative at the origin is energy independent and real leaving only one free parameter left which will be fixed below. In essence, this is the non-perturbative renormalization program with one counterterm described in [24] for singular potentials.

The ^{8}Be nucleus in its $(J^P, T) = (0^+, 0)$ ground state is unstable against α-decay, $^8\mathrm{Be} \to {}^4\mathrm{He} + {}^4\mathrm{He}$ with $Q = 91.84 \pm 0.04$KeV and a very small (Breit-Wigner) width $\Gamma_{BW} = 5.57 \pm 0.25$eV (see [25] for a review). The relative CM momentum is $p = \sqrt{2\mu_{\alpha\alpha} Q} = 19.2$MeV $(2\mu_{\alpha\alpha} = M_\alpha)$. The corresponding de Broglie wavelength, $\lambda \sim 10$fm, is much larger than the size of the α particle, so one would not expect internal structure playing a crucial role. The outer classical turning point is determined by the Coulomb potential yielding $r_{\max} = 62.6$fm. The situation is illustrated in Fig. 2 where the total $l = 0$ TPE plus the Coulomb potential barrier for the $\alpha - \alpha$ system are depicted, together with the experimental Q-value $Q = 91.84$KeV for the $^8\mathrm{Be} \to {}^4\mathrm{He} + {}^4\mathrm{He}$ reaction which proceeds by standard tunnel effect.

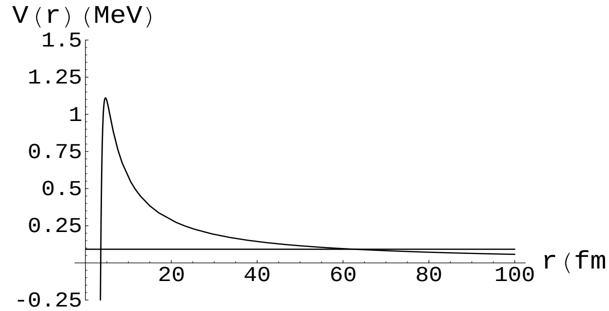

FIG. 2: The $l = 0$ total two-pion exchange plus Coulomb potential barrier for the $\alpha - \alpha$ system (in MeV) as a function of the relative distance (in fm). The horizontal straight line represents the energy corresponding to the energy $Q = 91.84$KeV of ^{8}Be nucleus in its ground state, $J^P = 0^+$.

It is instructive to analyze the decay process within the WKB approximation where the width is given by [26]

$$\Gamma_{\mathrm{WKB}} = \frac{e^{-2\int_{r_{\min}}^{r_{\max}} dr |p(r)|}}{4\mu_{\alpha\alpha} \int_{r_c}^{r_{\min}} \frac{dr}{2p(r)}} \, , \tag{15}$$

with $p(r) = \sqrt{2\mu_{\alpha\alpha}(Q - V(r))}$. Here, $r_{\min}$ and $r_{\max}$ are the classical turning points fulfilling $V(r_{\min}) = V(r_{\max}) = Q = M_{^8Be} - 2M_\alpha$. We obtain $\Gamma_{\mathrm{WKB}} = 8.6(4)$eV for the experimental Q, a correct order of magnitude compatible with the expected accuracy of the WKB formula. Although the TPE potential diverges at short distances, the inner classical turning point takes typically the value $r_{\mathrm{rmin}} = 3$fm for which there is about 1MeV cancellation between TPE and Coulomb potentials. So, the tunneling region is not determined by the singularity. Finite cut-off corrections to Γ_{WKB} are $\mathcal{O}(r_c^{9/2})$ for $r_c \ll r_{\min}$ as can be seen from Eq. (15).

A rigurous treatment of ^{8}Be as a exponentially time-decaying state requires finding a pole of the S-matrix in the second Riemann sheet of the complex energy plane, so we look for exact numerical solutions of Eq. (12) fulfilling the asymptotic boundary condition of a spherically outgoing Coulomb wave,

$$u_{0,p}(r) \to G_0(\eta,\rho) + i F_0(\eta,\rho) \, , \tag{16}$$

with $\eta = 1/(pa_B)$ and $\rho = pr$. For complex momenta $p = p_R + ip_I$ the energy also becomes complex $E = Q - i\Gamma/2$. The boundary condition, Eq. (14), implementing self-adjointness provides a correlation between Γ and Q through the TPE potential. We get $\Gamma_{\rm pole} = 3.4(2)\,{\rm eV}$ for the S-matrix pole width, fairly independently of the cut-off radius for $r_c \ll r_{\rm min} \sim 3$fm.

The experimentally determined [25] Breit-Wigner small width involves the s-wave phase shift [27, 28] $\Gamma_{\rm BW} = 2/\delta_0'(E_R)$ for $\delta_0(E_R) = \pi/2$. We get

$$\Gamma_{\rm BW}(^8{\rm Be} \to \alpha\alpha) = 4.3(3)\,{\rm eV}\,,\ (\,{\rm exp.}5.57(25){\rm eV}\,)\,, \quad (17)$$

for $E_R = 91.8$KeV and the couplings g_0, g_1 and g_2 with their uncertainties obtained from low energy $\pi\alpha$ scattering, Eq. (3) and [16]. Further, we analyze the scattering length α_0 and the effective range r_0 defined from the s-wave phase shift low energy expansion

$$\frac{2\pi \cot \delta_0(p)}{a_B(e^{2\pi\eta}-1)} + \frac{2}{a_B} h(\eta) = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \dots \quad (18)$$

with $h(x)$ the Landau-Smorodinsky function [27, 28]. From the universal low energy theorem of Ref. [24] in the Coulomb case we obtain (in fm)

$$r_0 = 1.03(1) - \frac{5.3(3)}{\alpha_0} + \frac{29(4)}{\alpha_0^2}\,. \quad (19)$$

The numerical coefficients depend on the total long distance potential (11) only. Using Eq. (14) we find $\alpha_0^{\rm th} = -1210(70)$fm and $r_0^{\rm th} = 1.03(1)$fm, in reasonable agreement with $\alpha_0 = -1630(150)$fm and $r_0 = 1.08(1)$fm from a low energy analysis of the data [27]. The correlation between (Q,Γ) and (α_0, r_0) was pointed out long ago [28, 29] but has no predictive power. As we see, this is compatible with the underlying chiral TPE potential which, in addition, correlates r_0 with α_0 and Γ with Q. At higher energies further ingredients are needed; for $E_{\rm LAB} = 3$MeV, i.e. $1/p \sim 2.5$fm, we get $\delta_0 = 141 \pm 2^0$ whereas $\delta_0 = 128.4 \pm 1^0$ from the scattering data [2]. Note that we have made no attempt to refit the couplings g_0, g_1 and g_2 stemming from low energy $\pi\alpha$ scattering, although it is obvious that further attraction is needed (a $20 - 30\%$ increase in g_0 would do). The fact that we produce numbers in the right order of magnitude is encouraging and calls for further improvements.

A more systematic determination of the input parameters would be highly desirable, possibly including pion loops in $\pi\alpha$ scattering. Moreover, an accurate description of $\alpha\alpha$ scattering data in the higher energy elastic region below $p+^7$Li and $n+^7$B production threshold or α-decays of the excited ^{8}Be states becomes sensitive to deeper interaction regions. This may require some peripheral nuclear structure information involving e.g. $\alpha \to t + p$ and $\alpha \to ^3{\rm He} + n$ intermediate state contributions on top of the chiral interactions deduced here. With these provisos in mind, the present work provides an example on how chiral symmetry might be useful in low energy nuclear reactions eventually providing theoretical constraints in the hardly accessible regions of astrophysical interest.

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[1] C. Rolfs and H. P. Trautvetter, Ann. Rev. Nucl. Part. Sci. **28**, 115 (1978).
[2] S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. **41**, 247 (1969).
[3] S. Weinberg, Physica **A96**, 327 (1979).
[4] T. E. O. Ericson and W. Weise, *Pions and Nuclei* (Oxford, UK: Clarendon (1988), 1988).
[5] P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002), nucl-th/0203055.
[6] I. Shimodaya, R. Tamagaki, and H. Tanaka, Prog. Theor. Phys. **27**, 793 (1962).
[7] S. R. Fang-Landau and M. P. Locher, Nucl. Phys. **B66**, 210 (1973).
[8] M. Ericson, P. Guichon, and R. D. Viollier, Nucl. Phys. **A372**, 377 (1981).
[9] C. A. Bertulani, H. W. Hammer, and U. Van Kolck, Nucl. Phys. **A712**, 37 (2002), nucl-th/0205063.
[10] M. P. Locher and T. Mizutani, Phys. Rept. **46**, 43 (1978).
[11] J. E. Beam, Phys. Rev. **158**, 907 (1967).
[12] E. E. Jenkins and A. V. Manohar, Phys. Lett. **B259**, 353 (1991).
[13] V. Bernard, N. Kaiser, and U.-G. Meissner, Int. J. Mod. Phys. **E4**, 193 (1995), hep-ph/9501384.
[14] G. Backenstoss, Ann.Rev.Nucl.Part.Sci. **20**, 467 (1970).
[15] H. Pilkuhn, N. Zovko, and H. G. Schlaile, Z. Phys. **A279**, 283 (1976).
[16] M. K. Khankhasaev, Nucl. Phys. **A505**, 717 (1989).
[17] B. Brinkmoeller and H. G. Schlaile, Phys. Rev. **C48**, 1973 (1993).
[18] K. P. Lohs, Nucl. Phys. **A312**, 297 (1978).
[19] N. Kaiser, Phys. Rev. **C64**, 057001 (2001).
[20] H. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[21] G. Feinberg, J. Sucher, and C. K. Au, Phys. Rept. **180**, 83 (1989).
[22] K. M. Case, Phys. Rev. **80**, 797 (1950).
[23] W. Frank, D. J. Land, and R. M. Spector, Rev. Mod. Phys. **43**, 36 (1971).
[24] M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. **C74**, 054001 (2006), nucl-th/0506047.
[25] D. R. Tilley et al., Nucl. Phys. **A745**, 155 (2004).
[26] S. A. Gurvitz and G. Kaelbermann, Phys. Rev. Lett. **59**, 262 (1987).
[27] G. Rasche, Nucl. Phys. **A94**, 301 (1967).
[28] M. W. Kermode, Nucl. Phys. **A104**, 49 (1967).
[29] M. W. Kermode, Nucl. Phys. **A134**, 336 (1969).